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COMPLEMENTED SUBSPACES OF $(l_2 \oplus l_2 \oplus \cdots)_p$ $(1 < p < \infty)$ WITH AN UNCONDITIONAL BASIS†

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ABSTRACT

It is proved that every infinite dimensional complemented subspace of $(l_2 \oplus l_2 \oplus \cdots)_p$ $(1 < p < \infty)$ with an unconditional basis is isomorphic to one of the following four spaces: l_2, l_p, $l_2 \oplus l_p$, $(l_2 \oplus l_2 \oplus \cdots)_p$.

1. Introduction

One of the basic questions in the isomorphic theory of Banach spaces is the following: Given a Banach space X, characterize those infinite dimensional Banach spaces which are isomorphic to a complemented subspace of X.

The answer to this question is known only in few cases, namely when X is isomorphic to c_0, $l_p (1 \leqq p \leqq \infty)$ ([7], [4]) or $l_{p_1} \oplus l_{p_2} \oplus \cdots \oplus l_{p_n}$ $(p_i \in [1,\infty) \cup \{0\}$, $1 \leqq i \leqq n$, $l_0 = c_0)$ ([2]).

The purpose of this paper is to deal with this question for $X = (l_2 \oplus l_2 \oplus \cdots)_p$ $(1 < p < \infty)$. The interest in studying this space stems from the fact that it is a complemented subspace of both L_p and C_p (see [1] for definition and properties of C_p).

It is very easy to check that l_2, l_p, $l_2 \oplus l_p$ and $(l_2 \oplus l_2 \oplus \cdots)_p$ are isomorphic to complemented subspaces of $(l_2 \oplus l_2 \oplus \cdots)_p$. It seems that there are no more isomorphic types of complemented subspaces of $(l_2 \oplus l_2 \oplus \cdots)_p$. In this paper we show that this is the case if we consider only subspaces with unconditional basis. Recall in this connection that it is still an open problem whether there are complemented subspaces of spaces with an unconditional basis which fail to have such a basis.

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THEOREM. *Let X be an infinite dimensional complemented subspace of $(l_2\oplus l_2\oplus\cdots)_p$ $(1<p<\infty)$ with an unconditional basis. Then, X is isomorphic to one of the following four spaces: l_2, l_p, $l_2\oplus l_p$ or $(l_2\oplus l_2\oplus\cdots)_p$.*

The proof of the theorem will be carried out in two steps. First, in Section 2, we shall make a reduction to the special case where X has a basis whose elements are disjointly supported with respect to the usual basis of $(l_2\oplus l_2\oplus\cdots)_p$. Then, in Section 3, we shall prove the theorem for this special case.

The notations are standard and those which are not explained here can be found in [5].

The usual basis of $(l_2\oplus l_2\oplus\cdots)_p$ will be denoted by $(e_{i,j})_{i,j=1}^{\infty}$, i.e., $e_{i,j}=(0,\cdots,0,f_j,0,\cdots,0)$ (f_j stands in the ith place), where f_j is the jth unit vector in l_2.

Thus, for all n, $m\in N$ and scalars $(a_{i,j})_{i=1,j=1}^{n\ \ m}$

$$\left\|\sum_{i=1}^{n}\sum_{j=1}^{m}a_{i,j}e_{i,j}\right\|=\left(\sum_{i=1}^{n}\left(\sum_{j=1}^{m}a_{i,j}^2\right)^{p/2}\right)^{1/p}.$$

We denote by Q_n, P_n and P_n^k, $n,k=1,2,\cdots$ the projections in $(l_2\oplus l_2\oplus\cdots)_p$ given by:

$$Q_n\left(\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}a_{i,j}e_{i,j}\right)=\sum_{i=1}^{n}\sum_{j=1}^{\infty}a_{i,j}e_{i,j}$$

$$P_n\left(\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}a_{i,j}e_{i,j}\right)=\sum_{j=1}^{\infty}a_{n,j}e_{n,j}$$

$$P_n^k\left(\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}a_{i,j}e_{i,j}\right)=\sum_{j=1}^{k}a_{n,j}e_{n,j}.$$

If X is a subspace of $L_p=L_p(0,1)$, then $X(l_2)$ denotes the Banach space of all sequences $\bar{f}=(f_1,f_2,\cdots)$ of functions on $(0,1)$ s.t. $f_i\in X$ for every $i=1,2,\cdots$ with respect to coordinatewise addition and scalar multiplication, and the norm:

$$\|\bar{f}\|=\left(\int_0^1\left(\sum_{i=1}^{\infty}f_i^2(t)\right)^{p/2}dt\right)^{1/p}=\left\|\left(\sum_{i=1}^{\infty}f_i^2(\cdot)\right)^{1/2}\right\|_{L_p}.$$

The sequence of the Rademacher functions on $[0,1]$ will be denoted by $(r_i)_{i=1}^{\infty}$.

2. Reduction to a special case

LEMMA 1. ([11], p. 224). *Let X, Y be subspaces of L_p $(1\leqq p<\infty)$ and let $T: X\to Y$ be a bounded linear operator. Then $\tilde{T}: X(l_2)\to Y(l_2)$ defined by:*

$$\tilde{T}((f_1, f_2, \cdots)) = (Tf_1, Tf_2, \cdots) \qquad \forall (f_1, f_2, \cdots) \in X(l_2)$$

is bounded (indeed $\|\tilde{T}\| = \|T\|$).

LEMMA 2. *Let* $(x_i)_{i=1}^{\infty}$ *be an unconditional basic sequence in* L_p $(1 \leqq p < \infty)$. *For* $i, j = 1, 2, \cdots$ *put*

$$\bar{x}_i^j = (0, \cdots, 0, x_i, 0, \cdots, 0)$$

(x_i *stands in the j-th place*).

Then $(\bar{x}_i^j)_{i,j=1}^{\infty}$ *is an unconditional basis for* $[x_i]_{i=1}^{\infty}(l_2)$ *and there exists a* $K > 0$ *s.t.*

$$K^{-1}\left\|\sum_{i,j=1}^{\infty} a_{i,j}\bar{x}_i^j\right\| \leqq \left\|\sum_{i=1}^{\infty}\left(\sum_{j=1}^{\infty} a_{i,j}^2\right)^{1/2} x_i\right\| \leqq K\left\|\sum_{i,j=1}^{\infty} a_{i,j}\bar{x}_i^j\right\| \tag{1}$$

for all sequences of scalars $(a_{i,j})_{i,j=1}^{\infty}$ *with only finitely many elements different from zero.*

This lemma was proved in [9] for the case $p > 2$; the proof here is much simpler.

We shall use the symbol $\approx$ to denote inequalities in both directions with constants which do not depend on the coefficients $(a_{i,j})_{i,j=1}^{\infty}$.

PROOF. It is easy to check that $(\bar{x}_i^j)_{i,j=1}^{\infty}$ spans $[x_i]_{i=1}^{\infty}(l_2)$, so it is sufficient to prove (1).

By Khinchine's inequality:

$$\left\|\sum_{i,j=1}^{\infty} a_{i,j}\bar{x}_i^j\right\|^p = \int_0^1 \left(\sum_{j=1}^{\infty}\left(\sum_{i=1}^{\infty} a_{i,j}x_i(t)\right)^2\right)^{p/2} dt$$

$$\approx \int_0^1\int_0^1 \left|\sum_{j=1}^{\infty} r_j(s)\sum_{i=1}^{\infty} a_{i,j}x_i(t)\right|^p dsdt.$$

Using the fact that $(x_i)_{i=1}^{\infty}$ is an unconditional basic sequence and the generalization of Khinchine's inequality for expressions of the form

$$\int_0^1\int_0^1 \left|\sum_{i,j=1}^{\infty} b_{i,j}r_i(u)r_j(s)\right|^p duds,$$

we get:

$$\left\|\sum_{i,j=1}^{\infty} a_{i,j}\bar{x}_i^j\right\|^p \approx \int_0^1\int_0^1\int_0^1 \left|\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{i,j}r_j(s)r_i(u)x_i(t)\right|^p dtduds$$

$$\approx \int_0^1 \left|\sum_{i,j=1}^{\infty} a_{i,j}^2 x_i^2(t)\right|^{p/2} dt$$

$$\approx \int_0^1 \left| \sum_{i=1}^{\infty} \left(\sum_{j=1}^{\infty} a_{i,j}^2 \right)^{1/2} x_i(t) \right|^p dt$$

(the last inequality follows from Khinchine's inequality and the unconditionality of $(x_i)_{i=1}^{\infty}$). ∎

LEMMA 3. *Let $(x_i)_{i=1}^{\infty}$ be an unconditional basic sequence in $(l_2 \oplus l_2 \oplus \cdots)_p$ $(1 \leqq p < \infty)$ so that $[x_i]_{i=1}^{\infty}$ is complemented in $(l_2 \oplus l_2 \oplus \cdots)_p$. Then there exists a sequence $(y_i)_{i=1}^{\infty}$ in $(l_2 \oplus l_2 \oplus \cdots)_p$ disjointly supported with respect to $(e_{i,j})_{i,j=1}^{\infty}$ such that $(y_i)_{i=1}^{\infty}$ is equivalent to $(x_i)_{i=1}^{\infty}$ and $[y_i]_{i=1}^{\infty}$ is complemented in $(l_2 \oplus l_2 \oplus \cdots)_p$.*

PROOF. Let T be the isomorphism from $(l_2 \oplus l_2 \oplus \cdots)_p$ into L_p defined by the requirement that $Te_{i,j}$ be the jth normalized Rademacher function supported on $[1-2^{-i+1}, 1-2^{-i})$ $(i,j = 1,2,\cdots)$.

By Lemmas 1 and 2 $[Tx_i]_{i=1}^{\infty}(l_2)$ is complemented in $(T((l_2 \oplus l_2 \oplus \cdots)_p))(l_2)$ and $(\overline{Tx_i^j})_{i,j=1}^{\infty}$ is an unconditional basis for $[Tx_i]_{i=1}^{\infty}(l_2)$ (where $\overline{Tx_i^j} = (0,\cdots,0,Tx_i,0,\cdots,0)$, Tx_i stands in the jth place). Thus, $[\overline{Tx_i^j}]_{i=1}^{\infty}$ is also complemented in $(T((l_2 \oplus l_2 \oplus \cdots)_p))(l_2)$.

It is easy to verify that $(\overline{Tx_i^j})_{i=1}^{\infty}$ is equivalent to $(Tx_i)_{i=1}^{\infty}$ and thus to $(x_i)_{i=1}^{\infty}$, and that $(\overline{Tx_i^j})_{i=1}^{\infty}$ is disjointly supported with respect to $(\overline{Te_{i,j}^k})_{i,j,k=1}^{\infty}$ (where $\overline{Te_{i,j}^k} = (0,\cdots,0,Te_{i,j},0,\cdots,0)$, $Te_{i,j}$ stands in the kth place).

Also, by Lemma 2,

$$\left\| \sum_{i,j,k=1}^{\infty} a_{i,j}^k \overline{Te_{i,j}^k} \right\| \approx \left(\sum_{i=1}^{\infty} \left(\sum_{j,k=1}^{\infty} (a_{i,j}^k)^2 \right)^{p/2} \right)^{1/p}$$

and thus $(\overline{Te_{i,j}^k})_{i,j,k=1}^{\infty}$ is equivalent to a permutation of $(e_{i,j})_{i,j=1}^{\infty}$. ∎

3. Proof of the theorem

The next lemma is well known and easy to prove (cf., e.g. [8]).

LEMMA 4. *Let $(x_i)_{i=1}^{\infty}$ be a normalized unconditional basic sequence in $(l_2 \oplus l_2 \oplus \cdots)_p$, then:*

(a) *If $2 < p < \infty$ $(x_i)_{i=1}^{\infty}$ is equivalent to the usual basis of l_2 iff there exists an $\varepsilon > 0$ and an integer $N \geqq 1$ s.t. $\|Q_N x_i\| > \varepsilon$ for $i = 1,2,\cdots$.*

(b) *If $1 \leqq p < 2$ and $(x_i)_{i=1}^{\infty}$ is equivalent to the usual basis of l_2, then for any $\varepsilon > 0$ there exists an integer $N \geqq 1$ s.t. $\|(I - Q_N)x_i\| < \varepsilon$ for $i = 1,2,\cdots$.*

Before we prove the theorem we deal first with a special case.

LEMMA 5. *Let $(x_i)_{i=1}^{\infty}$ be a normalized unconditional basic sequence in $(l_2 \oplus l_2 \oplus \cdots)_p$ $(1 < p < \infty)$ such that $[x_i]_{i=1}^{\infty}$ is complemented in $(l_2 \oplus l_2 \oplus \cdots)_p$ and such that no subsequence of $(x_i)_{i=1}^{\infty}$ is equivalent to the usual basis of l_2. Then $[x_i]_{i=1}^{\infty}$ is isomorphic to l_p.*

PROOF. For $p = 2$ the lemma is trivial, thus, by duality it is sufficient to give a proof for the case $2 < p < \infty$.

By Lemma 3 we can assume without loss of generality that $(x_i)_{i=1}^{\infty}$ is disjointly supported with respect to $(e_{i,j})_{i,j=1}^{\infty}$).

Let $\varepsilon > 0$. By the assumptions and Lemma 4 for each n there exist only finitely many indices i such that $\|P_n x_i\| \geqq 2^{-n} \cdot \varepsilon$ and thus, for each n there exists an integer $k_n \geqq 1$ s.t. $\|(P_n - P_n^{k_n})x_i\| < 2^{-n} \cdot \varepsilon$ for all $i = 1, 2, \cdots$.

Let us denote $Q = \sum_{n=1}^{\infty} P_n^{k_n}$.

Q is a norm one projection and $(Qx_i)_{i=1}^{\infty}$, $((I - Q)x_i)_{i=1}^{\infty}$ both have unconditionality constant one. (Here we use the assumption that $(x_i)_{i=1}^{\infty}$ are disjointly supported with respect to $(e_{i,j})_{i,j=1}^{\infty}$.

We shall show now that if ε is small enough then $(x_i)_{i=1}^{\infty}$ and $(Qx_i)_{i=1}^{\infty}$ are equivalent and thus, $[x]_{i=1}^{\infty}$ being an $\mathcal{L}_p$ space isomorphic to a subspace of $Q((l_2 \oplus l_2 \oplus \cdots)_p)$ (which is isomorphic to l_p), is isomorphic to l_p (by [3]).

It is clear that if $\sum_{i=1}^{\infty} \lambda_i x_i$ converges then $\sum_{i=1}^{\infty} \lambda_i Q x_i$ converges, so it is sufficient to show that if $\sum_{i=1}^{\infty} \lambda_i Q x_i$ converges then $\sum_{i=1}^{\infty} \lambda_i (I - Q) x_i$ converges.

The proof of this last fact is an imitation of a proof in [6].

Let $P: (l_2 \oplus l_2 \oplus \cdots)_p \xrightarrow{\text{onto}} X = [x_i]_{i=1}^{\infty}$ be the given projection and let $x_i^* \in (l_2 \oplus l_2 \oplus \cdots)_p$, $(p^{-1} + q^{-1} = 1)$ be given by:

$$Px = \sum_{i=1}^{\infty} x_i^*(x) x_i, \qquad \forall x \in (l_2 \oplus l_2 \oplus \cdots)_p. \tag{2}$$

Clearly

$$(I - Q)PQx_n = \sum_{i=1}^{\infty} x_i^*(Qx_n)(I - Q)x_i, \qquad n = 1, 2, \cdots.$$

The operator $(I - Q)P$ can be regarded as an operator from $[Qx_n]_{n=1}^{\infty}$ into $[(I - Q)x_n]_{n=1}^{\infty}$. The sequences $(Qx_n)_{n=1}^{\infty}$ and $((I - Q)x_n)_{n=1}^{\infty}$ are both unconditional basic sequences. Hence the diagonal operator defined by:

$$DQx_n = x_n^*(Qx_n)(I - Q)x_n, \qquad n = 1, 2, \cdots$$

is bounded ([10] or [5] p. 22).

Thus, the convergence of $\Sigma_{n=1}^{\infty}\lambda_n Qx_n$ implies the convergence of $\Sigma_{n=1}^{\infty}\lambda_n x_n^*(Qx_n)\,(I-Q)x_n$ and it is sufficient to show that $\inf_n |x_n^*(Qx_n)|>0$.

Now,

$$x_n^*(Qx_n)+x_n^*((I-Q)x_n)=x_n^*(x_n)=1, \qquad n=1,2,\cdots,$$

$$|x_n^*(I-Q)x_n|\leqq\|P(I-Q)x_n\|\leqq\|P\|\sum_{j=1}^{\infty}2^{-j}\cdot\varepsilon<\tfrac{1}{2}, \qquad n=1,2,\cdots,$$

if $\varepsilon<\|P\|^{-1}$. Thus, $|x_n^*(Qx_n)|>\frac{1}{2}$, $n=1,2,\cdots$. ■

REMARK 1. More careful examination of the proof of Lemma 3 shows that in the proof of the last lemma one can assume without loss of generality that x_n^* annihilates the support of x_m for $n\neq m$.

Thus, if ε is small enough, $Rx=\Sigma_{i=1}^{\infty}x_i^*(x)(x_i^*(Qx_i))^{-1}Qx_i$ defines a bounded projection $R:(l_2\oplus l_2\oplus\cdots)_p\xrightarrow{\text{onto}}[Qx_i]_{i=1}^{\infty}$; in particular, $[Qx_i]_{i=1}^{\infty}$ is complemented in $Q((l_2\oplus l_2\oplus\cdots)_p)$ and thus is isomorphic to l_p. There is therefore no need to use here the results of [3].

REMARK 2. Notice that in the assumptions of Lemma 5 $(l_2\oplus l_2\oplus\cdots)_p$ cannot be replaced by L_p, nor can (at least for $p>2$) the assumption that $[x_i]_{i=1}^{\infty}$ is complemented be dropped. The counterexample to both these statements is the space X_p of Rosenthal (see [8]).

PROOF OF THE THEOREM. By duality and Lemma 3 one can assume without loss of generality that $1<p<2$ and that X has a normalized basis $(x_i)_{i=1}^{\infty}$ which is disjointly supported with respect to $(e_{i,j})_{i,j=1}^{\infty}$.

We shall assume in addition that $[x_i]_{i=1}^{\infty}=X$ is not isomorphic to l_2, l_p or $l_2\oplus l_p$.

Let $P:(l_2\oplus l_2\oplus\cdots)_p\xrightarrow{\text{onto}}X$ denote the given projection and let $(x_i^*)_{i=1}^{\infty}$ be given by (2).

Fix ε, $\delta>0$. First we build by induction a strictly increasing sequence of integers $1\leqq N_1<N_2<\cdots$ and a sequence $((x_i^j)_{i=1}^{\infty})_{j=1}^{\infty}$ of disjoint subsequences of $(x_i)_{i=1}^{\infty}$ such that:

(3) $$\|Q_{N_k}^*(x_i^{k+1})^*\|<\delta, \qquad i,k=1,2,\cdots,$$

(4) $$\|(I-Q_{N_k})x_i^k\|<\varepsilon, \qquad i,k=1,2,\cdots,$$

(5) $(x_i^k)_{i=1}^{\infty}$ is 2-equivalent to the usual basis of l_2, $\qquad k=1,2,\cdots$.

By Lemma 5 and the assumption that X is not isomorphic to l_p one can find a subsequence $(x_i^1)_{i=1}^\infty$ of $(x_i)_{i=1}^\infty$ which is equivalent to the usual basis of l_2. By [8], Lemma 10] one can assume (by passing to a subsequence) that $(x_i^1)_{i=1}^\infty$ is 2-equivalent to this basis.

By Lemma 4 there exists an N_1 such that $\|(I - Q_{N_1})x_i^1\| < \varepsilon$, $i = 1,2,\cdots$.

Put,

$$A_1 = \{i \mid x_i^* \notin ((x_j^1)^*)_{j=1}^\infty \text{ and } \|Q_{N_1}^* x_i^*\| < \delta\}.$$

Note that for each N, Q_N^* is (formally) the same projection as Q_N but is acting in $(l_2 \oplus l_2 \oplus \cdots)_q$, $(p^{-1} + q^{-1} = 1)$. Thus, by Lemma 4 $[x_i^*]_{i \notin A_1}$ and hence $[x_i]_{i \notin A_1}$ are isomorphic to l_2. By the assumption that $[x_i]_{i=1}^\infty$ is not isomorphic to $l_2 \oplus l_p$ or l_2 it follows that $(x_i)_{i \in A_1}$ contains a subsequence $(x_i^2)_{i=1}^\infty$ which is 2-equivalent to the usual basis of l_2.

It follows from the definition of A_1 that $\|Q_{N_1}^*(x_i^2)^*\| < \delta$, for $i = 1,2,\cdots$.

Assume that we have found $(x_i^k)_{i=1}^\infty$, $1 \leqq k \leqq l+1$, and $N_1 < N_2 < \cdots < N_l$, such that (3) and (4) are satisfied for $1 \leqq k \leqq l$, $i = 1,2,\cdots$ and (5) is satisfied for $1 \leqq k \leqq l+1$.

According to Lemma 4 one can find $N_{l+1} > N_l$ so that (4) holds for $k = l+1$.

Put,

$$A_{l+1} = \{i \mid x_i^* \notin ((x_j^k)^*)_{j=1,}^\infty{}_{k=1}^{l+1} \text{ and } \|Q_{N_{l+1}}^* x_i^*\| < \delta\}.$$

By the arguments as in the first step, $[x_i]_{i \in A_{l+1}}$ is not isomorphic to l_p and is infinite dimensional, thus there exists a subsequence $(x_i^{l+2})_{i=1}^\infty$ of $(x_i)_{i \in A_{l+1}}$ which is 2-equivalent to the usual basis of l_2. By the definition of A_{l+1} the sequence $(x_i^{l+2})_{i=1}^\infty$ satisfies (3) for $k = l+2$.

This completes the proof of the existence of disjoint subsequences $(x_i^j)_{i=1}^\infty$, $j = 1,2,\cdots$, of $(x_i)_{i=1}^\infty$ and a sequence $N_1 < N_2 < \cdots$ satisfying (3), (4) and (5).

By (3) we have for $i, k = 1,2,\cdots$

$$\begin{aligned} 1 = (x_i^{k+1})^*(x_i^{k+1}) = (Q_{N_k}^*(x_i^{k+1})^*)(x_i^{k+1}) + (x_i^{k+1})^*((I - Q_{N_k})x_i^{k+1}) \\ \leqq \delta + \|P\| \|(I - Q_{N_k})x_i^{k+1}\| \end{aligned}$$

and hence:

$$\|(I - Q_{N_k})x_i^{k+1}\| > (1-\delta)\|P\|^{-1}. \tag{6}$$

By (4) and (6) we get, if we take e.g. $\delta = \frac{1}{2}$, $\varepsilon = (4\|P\|)^{-1}$, that for $i = 1,2,\cdots$, $k = 2,3,\cdots$,

$$\|(Q_{N_k} - Q_{N_{k-1}})x_i^k\| \geqq \|(I - Q_{N_{k-1}})x_i^k\| - \|(I - Q_{N_k})x_i^k\| \geqq (4\|P\|)^{-1}. \tag{7}$$

Now, from the assumption that x_i, $i = 1, 2, \cdots$, are disjointly supported it follows that for any sequence of scalars $(a_{i,k})_{i=1,k=2}^{\infty\ \infty}$ with only finitely many elements different from zero:

$$\left\| \sum_{k=2}^{\infty} \sum_{i=1}^{\infty} a_{i,k} x_i^k \right\| \leqq \left(\sum_{k=2}^{\infty} \left\| \sum_{i=1}^{\infty} a_{i,k} x_i^k \right\|^p \right)^{1/p} \leqq 2 \left(\sum_{k=1}^{\infty} \left(\sum_{i=1}^{\infty} a_{i,k}^2 \right)^{p/2} \right)^{1/p}$$

and

$$\left\| \sum_{k=2}^{\infty} \sum_{i=1}^{\infty} a_{i,k} x_i^k \right\| \geqq \left\| \sum_{k=2}^{\infty} \sum_{i=1}^{\infty} a_{i,k} (Q_{N_k} - Q_{N_{k-1}}) x_i^k \right\|$$

$$= \left(\sum_{k=2}^{\infty} \left\| \sum_{i=1}^{\infty} a_{i,k} (Q_{N_k} - Q_{N_{k-1}}) x_i^k \right\|^p \right)^{1/p} \geqq (4\|P\|)^{-1} \left(\sum_{k=2}^{\infty} \left(\sum_{i=1}^{\infty} a_{i,k}^2 \right)^{p/2} \right)^{1/p}.$$

(We used the fact that, if y_i, $i = 1, 2, \cdots$, are disjointly supported elements in $(l_2 \oplus l_2 \oplus \cdots)_p$, $(1 < p < 2)$, then

$$\left(\sum_{i=1}^{\infty} \|y_i\|^2 \right)^{1/2} \leqq \left\| \sum_{i=1}^{\infty} y_i \right\| \leqq \left(\sum_{i=1}^{\infty} \|y_i\|^p \right)^{1/p}.)$$

Thus, X contains a complemented isomorph of $(l_2 \oplus l_2 \oplus \cdots)_p$. This, together with the trivial fact that $(l_2 \oplus l_2 \oplus \cdots)_p$ is isomorphic to $(\Sigma \oplus (l_2 \oplus l_2 \oplus \cdots)_p)_p$, implies via Pełczyński's decomposition method that X is isomorphic to $(l_2 \oplus l_2 \oplus \cdots)_p$.

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